**Math 10A with Professor Stankova**
**Worksheet, Discussion #8; Wednesday, 9/13/2017**
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# Inverse Trig Functions

## Example

1. Find the derivative of $\arccos(x)$.

**Solution:** We know that $\cos(\arccos(x)) = x$ and taking the derivative of both sides with respect to $x$ gives $\sin(\arccos(x)) \cdot \arccos'(x) = 1$ and so $\arccos'(x) = \dfrac{1}{\sin(\arccos(x))}$. In order to get rid of the arccos, we need to express sin in terms of cos, and we can do this by using the formula $\sin^2(x) + \cos^2(x) = 1$ so $\sin(x) = \sqrt{1 - \cos^2(x)}$. Thus, we have that

$$\frac{d}{dx}\arccos(x) = \frac{1}{\sin(\arccos(x))} = \frac{1}{\sqrt{1 - \cos^2(\arccos(x))}} = \frac{1}{\sqrt{1 - x^2}}.$$

2. Find the derivative of $\arctan(x)$.

**Solution:** Starting with $\tan(\arctan(x)) = x$, we have that $\sec^2(\arctan(x)) \cdot \arctan'(x) = 1$. We want to find $\sec^2(x) = \frac{1}{\cos^2(x)}$ in terms of $\tan(x) = \frac{\sin(x)}{\cos(x)}$. Notice that both have $\cos(x)$ in the denominator and see that

$$\sec^2(x) = \frac{1}{\cos^2(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = 1 + \tan^2(x),$$

and hence

$$\frac{d}{dx}\arctan(x) = \frac{1}{\sec^2(\arctan(x))} = \frac{1}{1 + \tan^2(\arctan(x))} = \frac{1}{1 + x^2}.$$

# Derivative Definition Problems

## Example

3. Find $\lim_{x\to 1} \dfrac{\ln x}{x-1}$.

**Solution:** Let $f(x) = \ln x$, then $f(1) = \ln 1 = 0$ and so

$$\lim_{x\to 1} \frac{\ln x}{x-1} = \lim_{x\to 1} \frac{f(x)-f(1)}{x-1} = f'(1) = \frac{1}{1} = 1.$$

## Problems

4. Find $\lim_{h\to 0} \dfrac{(x+h)^2 - x^2}{h}$.

**Solution:** Let $f(x) = x^2$, then

$$\lim_{h\to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x) = 2x.$$

5. Find $\lim_{h\to 0} \dfrac{(x+h)^4 - x^4}{h}$.

**Solution:** Let $f(x) = x^4$, then

$$\lim_{h\to 0} \frac{(x+h)^4 - x^4}{h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x) = 4x^3.$$

6. Find $\lim_{h\to 0} \dfrac{\tan(x+h) - \tan x}{h}$.

**Solution:** Let $f(x) = \tan x$, then

$$\lim_{h\to 0} \frac{\tan(x+h) - \tan x}{h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x) = \sec^2(x).$$

7. Find $\lim_{h\to 0} \dfrac{(2x+h)^3+3(x+h)-2x^3-3x}{h}$.

**Solution:** Let $f(x) = 2x^3 + 3x$, then

$$\lim_{h\to 0} \frac{(2x+h)^3+3(x+h)-2x^3-3x}{h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x) = 6x^2+3.$$

8. Find $\lim_{h\to 0} \dfrac{\sin(2(x+h)) - \sin(2x)}{h}$.

**Solution:** Let $f(x) = \sin(2x)$, then

$$\lim_{h\to 0} \frac{\sin(2(x+h)) - \sin(2x)}{h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x) = 2\cos(2x).$$

9. Find $\lim_{h\to 0} \dfrac{\cos((x+h)^2) - \cos(x^2)}{h}$.

**Solution:** Let $f(x) = \cos(x^2)$, then

$$\lim_{h\to 0} \frac{\cos((x+h)^2) - \cos(x^2)}{h} = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = f'(x) = -2x\sin(x^2).$$

10. Find $\lim_{x\to 0} \dfrac{\tan x}{x}$.

**Solution:** Let $f(x) = \tan x$. Then $f(0) = 0$ and so

$$\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0} = f'(0) = \sec^2(0) = 1.$$

11. Find $\lim_{x\to \pi} \dfrac{\sin x}{x-\pi}$.

**Solution:** Let $f(x) = \sin x$. Then $f(\pi) = \sin\pi = 0$. So

$$\lim_{x\to \pi} \frac{\sin x}{x-\pi} = \lim_{x\to \pi} \frac{f(x)-f(\pi)}{x-\pi} = f'(\pi) = \cos(\pi) = -1.$$

12. Find $\lim_{x \to 0} \frac{\sin x}{x}$.

**Solution:** Let $f(x) = \sin x$. Then $f(\pi) = \sin 0 = 0$. So

$$\lim_{x \to 0} \frac{\sin x}{x} = \lim_{x \to 0} \frac{f(x) - f(0)}{x - 0} = f'(0) = \cos(0) = 1.$$

13. Find $\lim_{x \to \pi/4} \frac{\cos x - \sqrt{2}/2}{x - \pi/4}$.

**Solution:** Let $f(x) = \cos x$. Then $f(\pi/4) = \sqrt{2}/2$. So

$$\lim_{x \to \pi/4} \frac{\cos x - \sqrt{2}/2}{x - \pi/4} = \lim_{x \to \pi/4} \frac{f(x) - f(\pi/4)}{x - \pi/4} = f'(\pi/4) = -\sin \pi/4 = \frac{-\sqrt{2}}{2}.$$

14. Find $\lim_{x \to \pi/3} \frac{\sin x - \sin(\pi/3)}{x - \pi/3}$.

**Solution:** Let $f(x) = \sin x$. So

$$\lim_{x \to \pi/3} \frac{\sin x - \sin(\pi/3)}{x - \pi/3} = \lim_{x \to \pi/3} \frac{f(x) - f(\pi/3)}{x - \pi/3} = f'(\pi/3) = \cos \pi/3 = \frac{1}{2}.$$

15. Find $\lim_{x \to \pi/3} \frac{\sin x - \sqrt{3}/2}{x - \pi/3}$.

**Solution:** Let $f(x) = \sin x$. Then $f(\pi/3) = \sqrt{3}/2$. So

$$\lim_{x \to \pi/3} \frac{\sin x - \sqrt{3}/2}{x - \pi/3} = \lim_{x \to \pi/3} \frac{f(x) - f(\pi/3)}{x - \pi/3} = f'(\pi/3) = \cos \pi/3 = \frac{1}{2}.$$